MATH 1650 TRANSFORMATIONS OF GRAPHS SUMMARY

Suppose we are given the graph of $y = f(x)$. We have three broad categories of transformations:

SHIFTS: Given the graph of $y = f(x)$, to obtain the graph of:

- $y = f(x + h)$, **subtract** h from every x-value. This results in a **horizontal shift**.
  - If $h > 0$, this results in a shift to the **left** by h units.
  - If $h < 0$, this results in a shift to the **right** by h units.
- $y = f(x) + k$, **add** k to every y-value. This results in a **vertical shift**.
  - If $k > 0$, this results in a shift **up** by k units.
  - If $k < 0$, this results in a shift **down** by k units.

SCALINGS: Given the graph of $y = f(x)$, to obtain the graph of:

- $y = f(bx)$, **divide** every x-value by b. This results in a **horizontal scaling**.
  - If $b > 1$, this results in a horizontal **shrink** or **compression** by a factor of b.
  - If $0 < b < 1$, this results in a horizontal **stretch** or **dilation** by a factor of $1/b$.
- $y = af(x)$, **multiply** every y-value by a. This results in a **vertical scaling**.
  - If $a > 1$, this results in a vertical **stretch** or **dilation** by a factor of a.
  - If $0 < a < 1$, this results in vertical **shrink** or **compression** by a factor of $1/a$.

REFLECTIONS: Given the graph of $y = f(x)$, to obtain the graph of:

- $y = f(-x)$, multiply (divide) every x-value by -1. This results in a **reflection** about the y-axis.
- $y = -f(x)$, multiply (divide) every y-value by -1. This results in a **reflection** about the x-axis.

BIG IDEAS:

- Algebraic changes made 'inside' the function (to the argument) affect x-values.
- Algebraic changes made 'outside' the function affect the y-values.
- When adjusting the x-values, we do every thing opposite – we're **solving** for the new inputs.
- When adjusting the y-values, we do everything as written – we're **simplifying** the new outputs.

IN GENERAL:

Given the graph of $y = f(x)$, to graph $y = af(bx + h) + k$:

- **STEP 1:** Subtract h from each of the x-coordinates of the points on the graph of f.
- **STEP 2:** Divide the x-coordinates of the points on the graph obtained in STEP 1 by b.
- **STEP 3:** Multiply the y-coordinates of the points on the graph obtained in STEP 2 by a.
- **STEP 4:** Add k to each of the y-coordinates of the points on the graph obtained in STEP 3.

NOTE: To use this result, you will often need to rewrite given functions like in the following example:

$$y = \frac{1 - 2\sqrt{3 - 4x}}{5} \longleftrightarrow y = -\frac{2}{5}\sqrt{-4x + 3} + \frac{1}{5}$$